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ULTIMATE VELOCITIES OF PLATES ACCELERATED BY MAGNETIC FIELD

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The problem on acceleration of conductive plates by a magnetic field has been extensively studied. The ultimate velocities obtained during induction launching and railgun acceleration of metallic bodies were studied analytically and numerically [1-6]. Without considering the details, we note that an analytic description was carried out in these papers with a large number of simplifying assumptions which narrow the area of application of the results. At the same time the results obtained using sufficiently complete numerical models have an even narrower area of application by virtue of the bounded set of initial data governing the acceleration dynamics, which makes impossible any conclusions on ultimate potentialities of the acceleration of solid bodies by a magnetic field.

In the present paper the ultimate kinematic characteristics of one- and multi-layer conductive plates accelerated by a nonstationary magnetic field are studied, as well as their dependence on accelerated mass and acceleration distance. One- and two-dimensional problems as applied to induction and railgun accelerators of conductive projectiles are considered. For the one-dimensional case the equations relating maximum admissible velocities of the plate and kinetic energy to accelerated mass and acceleration distance are derived.

1. Statement of the Problem. Let us consider the acceleration of a conductive plate of finite thickness d by the pressure of a nonstationary magnetic field $H_0(t)$. The plate consists of N layers of different metals of thicknesses $\Delta x_i(\Sigma^N_{i=1}\Delta x_i = d)$. One edge of the plate slides down the surface of current-carrying railgun electrode in the direction X (Fig. 1) with velocity V(t), thus forming an ideal metallic contact with the surface of the rail Γ_4. Let us assume that the plate size is unlimited in the direction $-y$, while the rail is unlimited in the directions $-x$ and x. Moreover, the entire system is infinite in the direction z, which is perpendicular to the plane of the figure, i.e., the magnetic field in this problem has one component H_z (hereafter the subscript z will be omitted) and depends only on two coordinates x and y.

During acceleration the magnetic field gradually diffuses in the plate, as a result of which its surface temperature T increases and can exceed the melting point or even the temperature of vaporization of the i-layer. In this case the projectile can fail, and/or a gap in the metallic contact can occur when passing to the acceleration regime with electric arc contact. Let us assume that during acceleration the maximum values of temperature in each layer should not exceed the critical values T_i^* (melting or vaporation temperatures) for these layers. Let us term the velocity of the plate at which the temperature achieves its critical value in a certain layer the ultimate velocity for the given acceleration regime. In calculating the temperature distribution in the plate and rail we take into account the energy transfer due to the heat conductivity and assume that each of the subdomains Ω_i of the problem is characterized by constant values of density ρ, heat capacity c_i, heat conductivity k_i, and electric conductivity σ_i.

The system of differential equations written in the reference system with respect to the moving conductive plate describes the problem stated

$$\mu\left(\frac{\partial H}{\partial t} - V\frac{\partial H}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{1}{\sigma}\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\sigma}\frac{\partial H}{\partial y}\right),$$
$$\rho c\left(\frac{\partial T}{\partial t} - V\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{1}{\sigma}\left(\left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2\right). \quad (1.1)$$

The boundary and initial conditions have the form

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$$\left.\frac{\partial H}{\partial n}\right|_{\Gamma_5} = H\Big|_{\Gamma_{3,6,7}} = \left.\frac{\partial T}{\partial n}\right|_{\Gamma_{1,2,3,5,6,7}} = 0, \quad H\Big|_{\Gamma_1} = H_0(t), \tag{1.2}$$

$$T(x, y, 0) = T_0, \quad H(x, y, 0) = 0.$$

In Eqs. (1.1) the velocity V is nonzero only in the region of rails $y \geq 0$; ρ, c, k, σ are step functions of x and y; μ is the magnetic permeability of vacuum. At the inner boundaries of the layers B_i and contact boundary Γ_4, where the medium properties are discontinuous, the following conjunction conditions should be given: the continuity of temperatures, magnetic fields, heat flux, and tangential constituents of electric fields, i.e.,

$$[T] = [H] = \left[k\frac{\partial T}{\partial n}\right] = \left[\frac{1}{\sigma}\frac{\partial H}{\partial n}\right] = 0 \text{ for } \Gamma_4, B_i, i = 1, ..., N-1. \tag{1.3}$$

Here and in (1.2) n is the unit vector normal to the boundaries Γ_i and B_i. The equations of motion for the projectile may be written as follows:

$$M\frac{dV}{dt} = \frac{\mu H_0^2(t)}{2}, \quad V = \frac{dL}{dt}; \tag{1.4}$$

$$M = \sum_{i=1}^{N} \rho_i \Delta x_i \tag{1.5}$$

(M is the projectile mass per unit-area and L is the acceleration distance.)

Let us write the system of equations (1.1)-(1.5) in dimensionless form, introducing the dimensionless variables η, ξ, τ, θ, n, v, l, m, $\tilde{\sigma}$, $\tilde{\rho}$, $\tilde{c}$, and $\tilde{k}$ which are connected with the dimensional variables by the relationships

$$x = x_s\xi, \quad y = y_s\eta, \quad t = t_s\tau, \quad H = H_s h, \quad M = M_s m, \quad V = V_s v, \tag{1.6}$$
$$L = x_s l, \quad \sigma = \sigma_s\tilde{\sigma}, \quad \rho = \rho_s\tilde{\rho}, \quad c = c_s\tilde{c}, \quad k = k_s\tilde{k}, \quad T = T_0 + T_s\theta.$$

As the scales σ_s, ρ_s, c_s, k_s we select the values of these quantities at the first layer σ_1, ρ_1, c_1, k_1. Furthermore, if we assume that the following correlations between the scales of transformations H_s, T_s, x_s, V_s, t_s, M_s hold,

$$\frac{\mu H_s^2}{\rho_1 c_1 T_s} = 1, \quad \frac{t_s}{\mu\sigma_1 x_s^2} = 1, \quad \frac{M_s}{\rho_1 x_s} = 1, \quad \frac{x_s}{t_s V_s} = 1, \quad \frac{\mu H_s^2}{2\rho_1 V_s^2} = v_0 \tag{1.7}$$

and denote $h_0(\tau) = H_0(t_s\tau)/H_s$ and $\gamma = k_1\mu_1\sigma_1/\rho_1 c_1$, then the system of equations (1.1)-(1.5) can be transformed to the dimensionless system of differential equations

$$\frac{\partial h}{\partial \tau} - v\frac{\partial h}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{1}{\tilde{\sigma}}\left(\frac{\partial h}{\partial \xi}\right)\right) + \frac{\partial}{\partial \eta}\left(\frac{1}{\tilde{\sigma}}\left(\frac{\partial h}{\partial \eta}\right)\right), \tag{1.8}$$

$$\tilde{\rho}\,\tilde{c}\left(\frac{\partial \theta}{\partial \tau} - v\frac{\partial \theta}{\partial \xi}\right) = \frac{\partial}{\partial \xi}\left(\tilde{k}\gamma\left(\frac{\partial \theta}{\partial \xi}\right)\right) + \frac{\partial}{\partial \eta}\left(\tilde{k}\gamma\left(\frac{\partial \theta}{\partial \eta}\right)\right) + \frac{1}{\tilde{\sigma}}\left(\left(\frac{\partial h}{\partial \xi}\right)^2 + \left(\frac{\partial h}{\partial \eta}\right)^2\right).$$

Here

$$v = \frac{v_0}{m}\int_0^\tau h_0^2(\tau)d\tau, \quad l = \frac{v_0}{m}\int_0^\tau\left(\int_0^\tau h_0^2(\tau)d\tau\right)d\tau, \quad m = \sum_{i=1}^{N}\tilde{\rho}_i\Delta\xi_i. \tag{1.9}$$

The boundary conditions and the conditions of the field conjunction at the boundaries are analogous to (1.2) and (1.3). The parameter ν_0 in the last relationship of (1.7) can have an arbitrary value (including unitary one) and will be used hereinafter only to simplify the transition to the one-dimensional problem.

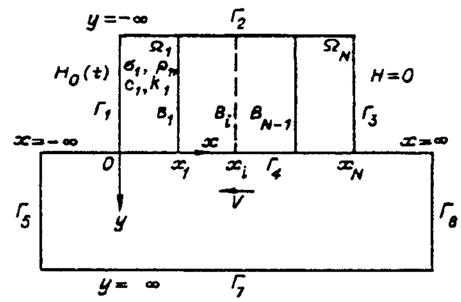

Fig. 1

The six scale coefficients (H_s, T_s, x_s, V_s, t_s, M_s) are connected by five equations; consequently, usually only one coefficient can be specified arbitrarily, and all the others can be expressed in terms of it, using the correlations (1.7). Let us take T_s as an arbitrary coefficient and choose a value of it such that for an arbitrary instant of time τ' the solution of the system of equations (1.8) corresponds to a real physical process in which a value equal to the critical temperature of this layer T_i^* is achieved in a certain layer i at the instant of the $t' = t_s\tau'$, and for the other layers the following relationships hold:

$$T_s(\tau')\max_{\xi,\eta\in\Omega_i}(\theta(\xi, \eta, \tau)) \leq T_i^* - T_0, \quad i = 1, \ldots, N. \tag{1.10}$$

If we define the dimensionless critical temperature as

$$\theta^*(\tau') = \max_{i=1,N}\left\{\left(\frac{T_1^* - T_0}{T_i^* - T_0}\right)\max_{\xi,\eta\in\Omega_i}(\theta(\xi, \eta, \tau))\right\} \tag{1.11}$$

and the transformation scale for temperature as

$$T_s(\tau') = \frac{T_1^* - T_0}{\theta^*(\tau')} = \frac{\Delta T_1^*}{\theta^*(\tau')}, \tag{1.12}$$

then the correlations (1.10) will hold automatically. It should be noted that, by virtue of the definition (1.11), θ^* is a continuous function of τ, but only with a piecewise derivative.

Using (1.12), we express H_s, T_s, x_s, V_s, t_s, and M_s from (1.7) and substitute the relationships obtained in (1.6). After a series of transformations we obtain

$$H_0(\tau) = H_T h_0(\tau)/\theta^{*1/2}(\tau'); \tag{1.13}$$

$$V(\tau) = V_T(v_0\theta^*(\tau'))^{-1/2}v(\tau); \tag{1.14}$$

$$L(\tau) = \frac{(v_0\theta^*(\tau'))^{1/2}}{\mu\sigma_1 V_T} l(\tau); \tag{1.15}$$

$$M(\tau') = \frac{\rho_1 m(v_0\theta^*(\tau'))^{1/2}}{\mu\sigma_1 V_T}; \tag{1.16}$$

$$t(\tau) = \frac{v_0\theta^*(\tau')\tau}{\mu\sigma_1 V_T^2}; \tag{1.17}$$

$$T(\xi, \eta, \tau) = \Delta T_1^* \frac{\theta(\xi, \eta, \tau)}{\theta^*(\tau')} + T_0; \tag{1.18}$$

$$H(\xi, \eta, \tau) = H_T \frac{h(\xi, \eta, \tau)}{\theta^*(\tau')^{1/2}}, \tag{1.19}$$

where $H_T = (\rho_1 c_1 \Delta T_1^*/\mu)^{1/2}$, $V_T = (c_1 \Delta T_1^*/2)^{1/2}$.

Relationships (1.13)-(1.19) describe the variations in the appropriate physical values depending on dimensionless time τ, the temperature in any of the layers achieving the critical value at $\tau = \tau'$ by virtue of the definitions (1.11) and (1.12), while the velocity $V(\tau')$ will be the ultimate one up to which the conductive projectile with the mass per unit area $M(\tau')$ can be accelerated at distance $L(\tau')$.

2. One-Dimensional Case. Assume the induction acceleration of a conductive plate, i.e., there is no sliding contact. Then there will be neither terms with derivatives with respect to η nor terms with first derivative with respect to ξ in the system of equations (1.8). Let us assume the plate to be infinite in the directions y and z. Since in this case the velocity ν is not involved in Eq. (1.8), the function $\theta^*(\tau)$ will not depend on the transformation scale for the equations of motion ν_0, which allows us to exclude the latter from Eq. (1.14)-(1.18). Moreover, let us seek the ultimate velocity which the projectile acquires at a given distance; in correlations (1.13)-(1.19) we consider $\tau = \tau'$ and hereinafter omit the prime.

Substituting ν_0 derived from (1.15) into (1.14) and (1.16), we obtain

$$V = V_T^{4/3} (\mu\sigma_1 L)^{1/3} I_1 I_2^{-1/3}; \tag{2.1}$$

$$M = \rho_1 \left(\frac{L}{\mu^2 \sigma_1^2 V_T^2 I_2} \right)^{1/3}, \tag{2.2}$$

where

$$I_1 = \frac{\int_0^\tau h_0^2(\tau) d\tau}{m^2 \theta^*(\tau)}; \tag{2.3}$$

$$I_2 = \frac{\int_0^\tau \left(\int_0^\tau h_0^2(\tau) d\tau \right) d\tau}{m^4 \theta^*(\tau)}. \tag{2.4}$$

Note that in the case of the arbitrary function $h_0(\tau)$ the dimensionless critical temperature $\theta^*(\tau)$ involved in (2.3) and (2.4) depends on both τ and m; however, if the field at the boundary is given as the power function

$$h_0(\tau, n) = \tau^{n/2}, \tag{2.5}$$

then one can show that owing to invariance of the system (1.8) with respect to the transformation $\tau' = a^2\tau$, $\xi' = a\xi$, $h' = bh$, $\theta' = b^2\theta$ and self-similarity of the power function (i.e., $(ax)^n = a^n x^n$), the expressions (2.3) and (2.4) will depend only on the ratio τ/m^2. This allows us to derive the dependence V(M, n) with L = const using (2.1)-(2.4) from the known function $\theta^*(\tau)$ obtained from the solution of the system (1.8) for an arbitrary value m and the magnetic field at the boundary, which changes following the law (2.5).

Let us consider the acceleration of a homogeneous plate when the magnetic field at the boundary changes according to (2.5). In this case θ^* coincides with the maximum value of $\theta(\xi, \tau)$, $d\theta^*(\tau)/d\tau$ is a continuous function of τ, and $m = \Delta\xi_1$ may be chosen equal to one. Let us find the maximum dependence V(M). For this purpose we calculate the derivative dV/dM and set it equal to zero. Using (2.1)-(2.5), we obtain the equation for determining the time τ_0 corresponding to the maximum of the dependence V(M)

$$\tau \frac{d\theta^*}{d\tau} = \left(n + \frac{1}{2}\right)\theta^*. \tag{2.6}$$

let us introduce the notation

$$I_V(n) = I_1(\tau_0, n)I_2^{-1/3}(\tau_0, n), \quad I_M(n) = I_2^{-1/3}(\tau_0, n),$$
$$I_H(n) = h_0(\tau_0)/\theta^{*1/2}(\tau_0), \quad V_{max} = V(\tau_0), \quad M_{opt} = M(\tau_0),$$

then from (1.2) and (1.13) we obtain

$$V_{max} = V_T^{4/3}(\mu\sigma L)^{1/3} I_V(n); \tag{2.7}$$

$$M_{opt} = \rho\left(\frac{L}{\mu^2\sigma^2 V_T^2}\right)^{1/3} I_M(n); \tag{2.8}$$

$$H_{opt} = H_T I_H(n). \tag{2.9}$$

The quantities I_H, I_V, I_M depend on the parameter γ, which is equal to the ratio of the thermal conductivity to the coefficient of diffusion of the magnetic field in the metal [see (1.8)]. However, for the majority of metals this dependence can be neglected. The maximum relative error in this case is for copper, but it does not exceed 5% (Fig. 2).

The values of I_H, I_V, I_M for n = 1-5 obtained from numerical solution of the system (1.8) and Eq. (2.6) are presented in Table 1. It is apparent that the dependence on n, i.e., eventually on the shape of the current pulse in the case of induction acceleration of conductive plates, is also weak.

Table 2 presents the maximum values of ultimate velocity V_{max}, optimum magnetic field amplitude H_{opt}, specific kinetic energy E, optimum mass per unit area M_{opt}, and the appropriate thickness of the accelerated plate d for a number of metals at acceleration distance L = 1 m, n = 1, $\gamma = 0$, and critical temperature equal to the melting temperature of the given metal. The values are calculated from (2.7)-(2.9).

Figure 2 shows the dependence of ultimate velocity on mass for tungsten (curves 1-3) and copper (curves 4-6), as well as for copper when $\gamma = 0$, i.e., without regard for thermal conductivity (curves 7-9). Curves 1, 4, and 7 correspond to acceleration distance L = 0.5 m, curves 2, 5, and 8 are plotted for distance L = 1 m, and curves 3, 6, and 9 are for L = 2 m at n = 1.

It should be noted that with small M the dependence V(M) approaches asymptotically the straight line V/M = const. This corresponds to the solution of the problem of acceleration of conductive plates in the thin-plate approximation cited in [1]. The other limiting case corresponding to high values of M (thick-plate approximation) can be obtained by using the known solution of the problem of heating of a semi-infinite projectile by a current pulse given as the power function (2.5) with neglect of the heat conductivity [1]:

$$\theta^* = \theta(\tau)\Big|_{\xi=0} = \tau^n\varphi(n), \quad \varphi(n) = \frac{1}{n}\left(\Gamma(n/2 + 1)/\Gamma(n/2 + 1/2)\right)^2, \tag{2.10}$$

where $\Gamma(x)$ is the gamma function.

This case conforms to $M \to \infty$ and holds true if the time of acceleration of the plate is much shorter than that of diffusion of the magnetic field through the plate of thickness d ($t << \mu\sigma d^2$). Substituting (2.10) in (2.3) and (2.4), we obtain

$$I_1 = \frac{\tau}{m^2}\frac{1}{(n + 1)\varphi(n)}, \quad I_2 = \frac{\tau^2}{m^4}\frac{1}{(n + 1)(n + 2)\varphi(n)}. \tag{2.11}$$

Using (2.1) and (2.2) together with (2.11) and excluding the dependence on τ/m^2, we have

$$V_\infty = V_T(\rho L\psi(n)/M)^{1/2},$$

TABLE 1

n	l_H	l_V	l_M	ψ
1	0,951	1,58	0,927	1,91
2	1,046	1,66	0,938	2,09
3	1,075	1,67	0,930	2,12
4	1,087	1,67	0,923	2,12
5	1,093	1,66	0,917	2,11

TABLE 2

Metal	V_{max}, km/sec	M_{opt}, g/cm^2	d, mm	H_{opt}, kA/cm	E, kJ/cm^2
Be	30,9	0,26	1,42	514	125
Mg	13,5	0,43	2,48	288	39
Al	15,2	0,49	1,82	346	57
Ti	7,4	5,38	11,9	557	147
V	9,5	4,36	7,16	646	197
Cr	10,2	4,16	5,79	677	216
Mn	4,6	14,3	19,2	572	155
Fe	10,8	3,23	4,1	634	190
Co	11,7	2,78	3,16	637	192
Ni	11,5	2,8	3,34	629	187
Cu	14,1	1,33	1,49	531	133
Zr	5,5	7,61	11,7	497	117
Nb	9,6	4,36	5,05	655	203
Mo	12,5	3,07	3,01	710	238
Ag	9,5	1,81	1,72	416	82
La	2,5	12,8	20,8	287	39
Hf	4,3	15,7	11,8	556	146
Ta	7,1	9,76	5,87	719	244
W	10,0	6,18	3,22	807	308
Re	6,3	15,5	7,56	814	313
Os	7,0	11,4	5,04	770	280
Ir	7,9	7,95	3,55	729	250
Pt	5,2	12,9	6,01	608	174
Au	6,0	5,06	2,62	439	91
Th	3,8	11,2	9,57	416	82
U	2,5	28,1	14,8	431	88

where $\Psi(n) = (n + 2)/((n + 1)\varphi(n))$. In this case the ultimate kinetic energy E_∞ does not depend on M:

$$E_\infty = \frac{MV^2}{2} = \rho V_T^2 L\psi(n) = \frac{\rho c\Delta T^*}{2} L\psi(n). \tag{2.12}$$

The values of $\psi(n)$ for n = 1-5 are presented in Table 1. The kinetic energy determined by the expression (2.12), as follows from Fig. 3, is the maximum kinetic energy that can be acquired by a conductive plate during induction acceleration. Figure 3 presents, for tungsten, the dependences V(M) and E(M) (lines 1 and 4), the asymptotical dependences conforming to $V(M \to O)$ and $V(M \to \infty)$ (lines 2 and 3 correspond to the thin- and thick-plate approximation, respectively), and the value of the ultimate kinetic energy E_∞ (line 5) for L = 1 m, n = 1.

Let us consider some results of the solution of the problem on acceleration of bimetallic plates. Figure 4 shows the dependences of ultimate velocity vs mass for a tungsten–beryllium plate with L = 1 m, n = 1, and different mass ratios of the layers: 1) Be (100%), 2) Be (0), 3) Be (8.2%), 4) Be (25%), and 5) Be (36%). It is apparent that a considerable increase in maximum velocity is possible (in this case ≈60%); at the same time, the optimal choice of the thickness of the layers depends on the total mass of the plate, i.e., for each value M there is an approximate value M_{Be}/M_W at which the maximum velocity increment for the bimetallic plate is achieved. On the basis of these data, one can state the problem of search for the optimum combination of layers providing the ultimate velocity for a given total mass of the plate.

3. Two-Dimensional Case. Given the moving metallic contact of the plate with the rails, consider the solution of the system of equations (1.8). Let the field at the boundary be set as before by the function (2.5). In this case θ^* depends on ν_0; therefore, the latter cannot be excluded from (1.14)-(1.16). Given the transformation scale T_s, at any instant of time τ all scales are determined uniquely from (1.7). The system of equations (1.8) with fixed value ν_0 and various values of m_j should

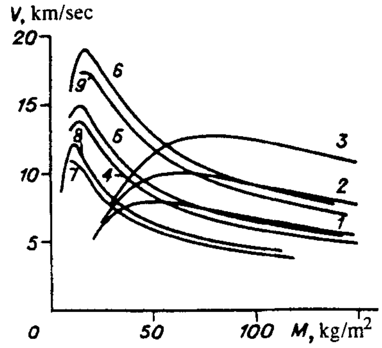

Fig. 2

be solved to derive the dependence of ultimate velocity on mass. Then, by eliminating τ from the obtained set of dependences $V(m_j, \tau)$, $M(m_j, \tau)$, $L(m_j, \tau)$, one can derive the dependences $V_j(L)$, $M_j(L)$ for L = const and interpolate them, for example, by constructing a spline function.

Figure 5 shows the dependences of ultimate velocity on mass for rails and projectiles made of tungsten (curve 2), and copper and tungsten respectively (curve 3) (here and in the other figures $L = 1$ m, $n = 1$). Curve 1, corresponding to induction acceleration of the tungsten plate, is presented for comparison. Given the moving contact, the considerable decrease of the maximum of ultimate velocity and optimum mass is apparent, although the general character of the dependence of ultimate velocity on mass is qualitatively the same as that obtained for the one-dimensional case. At $M \to 0$ both one- and two-dimensional dependences material. This is due to the fact that when accelerating very thin plates, volume heating of the projectile away from the zone of metallic contact appears to be predominant, since the increase in heat release resulting from the current concentration in the zone of moving metallic contact is balanced by heat transfer in the rails.

In the two-dimensional case the rate of heating of the projectile near the point with maximum temperature depends essentially on its heat conductivity, because this point is on the plate surface when $x = 0$ near the rails whose temperature at considerably high velocity differs little from the initial one. As a result, significant temperature gradients and, consequently, high-power heat flux from the projectile to the rail appear in this zone.

Figure 6 shows the effect of heat conductivity. Curves 1 and 3 represent the dependences of ultimate velocity on mass for tungsten ($k = k_W$) and copper ($k = k_{Cu}$), curve 2 is obtained for tungsten, but with $k = k_{Cu}$, and curve 4 is for Cu with $k = k_W$. In comparing lines 1 and 2 with 3 and 4, one can see the considerable influence of heat conductivity on ultimate velocity.

Figure 7 shows that in the case of acceleration of bimetallic projectiles in railguns the ultimate velocity can be increased. Here lines 1 and 2 correspond to acceleration of homogeneous projectiles of tungsten and beryllium, and curve 3 is for a bimetallic plate with mass ratio W 40% – Be 60%. In this case the maximum relative velocity increment (~60%) is achieved when the total mass of the plate is 7 kg/m^2. The dependences presented in Figs. 6 and 7 are obtained for copper electrodes.

The analysis of induction and railgun acceleration of metallic plates certainly does not take into account the many peculiarities of real physical processes like the nonlinear diffusion of the magnetic field, the three-dimensional character of distribution of the magnetic field leading to the reduction of effective magnetic pressure, the friction in the moving contact causing additional heat release in the contact zone, etc. Thus, the above absolute values of velocity and mass can differ considerably from those achieved in real launching. However, all the effects enumerated lead to a reduction of the final velocity. Therefore, the above results may be considered as the ultimate ones for any acceleration process realized in practice. Thus, if we require that the increase in temperature of the accelerated plate during the acceleration process due to Joule heating does not exceed the critical value(s) for acceleration of multi-layer projectiles, then we can make the following assertions.

For the one-dimensional case: 1) There is an ultimate velocity up to which a plate made of a given metal can be accelerated. This velocity depends mainly on the mass of the accelerated plate and weakly depends on the shape of the current pulse (at least for the case where the current pulse is described by a monotonically growing function). 2) There is an optimum plate thickness (or mass per unit area) at which the ultimate velocity achieves the maximum value. This maximum velocity and optimum mass may be calculated from the analytical relationships (2.7) and (2.8). 3) There is a limit of kinetic

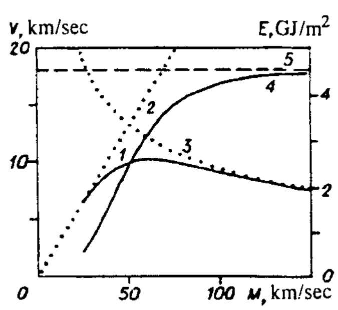

Fig. 3

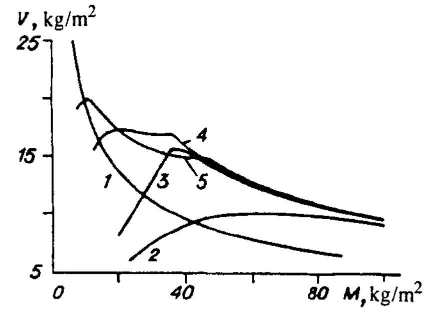

Fig.4

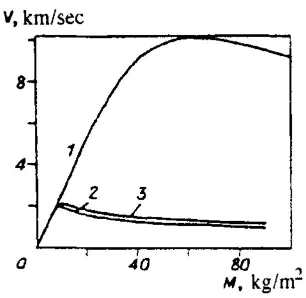

Fig. 5

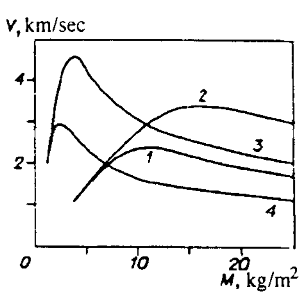

Fig. 6

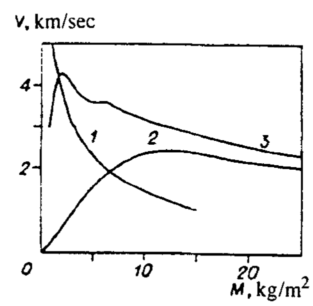

Fig. 7

energy per unit area which can be acquired by a plate. This limit is achieved when the plate mass exceeds the optimum value. 4) For plates made of different metals the values of optimum mass and maximum velocity can be considerably different, i.e., to achieve the maximum velocity the choice of the material for the plate should depend on the accelerated mass (see Table 2). 5) A considerable increase in the ultimate velocity for multi-layer plates is possible, and for maximum relative velocity gain the ratio of the layer thicknesses depends on the mass.

For the two-dimensional case: 1) All of the qualitative assertions made for induction acceleration hold true for the acceleration of metallic projectiles in railguns. 2) The presence of a moving metallic contact leads to a significant reduction of the maximum ultimate velocity and optimum mass. 3) For correct calculations of the temperature pattern in the zone of moving contact the heat transfer due to heat conductivity should be taken into account. 4) An increase in rail conductivity as compared to the projectile conductivity leads to an increase in the maximum values of the ultimate velocity, and at $\sigma_r \to \infty$ the results agree with those for induction acceleration.

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